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ABSTRACT

Fourteen research reports related to mathematics education are abstracted and analyzed. Five studies deal with aspects of mathematics instruction, and there are four reports each on mathematics achievement and on two cognitive processes. The remaining items concern calculators, attitudes, and aptitude-treatment interaction. Research related to mathematics education reported in CIJE and RIE between January and March 1982 is also noted. (MP)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Summer 1982

Arlin, Patricia Kennedy. PIAGETIAN TASKS AS PREDICTORS OF READING AND MATH READINESS IN GRADES K-1. Journal of Educational Psychology 73: 712-721; October 1981.

Abstract and comments prepared for I.M.E. by THOMAS P. CARPENTER, University of Wisconsin, Madison.

1. Purpose

The major purpose of the study was to investigate the relationship between performance on a series of nine Piagetian tasks of concrete operations and performance on standardized tests of reading and mathematics. A secondary purpose was to identify the order of acquisition of the nine Piagetian tasks.

2. Rationale

A number of studies have demonstrated that a significant positive correlation exists between individual Piagetian tasks and achievement in reading and mathematics. The development of concrete operations, however, involves the acquisition of three related but distinct subsystems: conservation, seriation, and classification. Studies that have focused on isolated skills provide an incomplete picture of development, and individual tasks may be somewhat limited in their usefulness as readiness measures. Consequently, none of the earlier studies completely justify the use of Piagetian tasks as measures of readiness. The fact that positive correlations have been reported between individual tasks and achievement does suggest, however, that a more complete battery of tasks that takes into account the entire "structure d' ensemble" of concrete operations may provide an appropriate measure of readiness in reading and mathematics.

3. Research Design and Procedures

One hundred ninety-two children from five schools in a large suburban school district were individually tested on a set of nine Piagetian tasks at the end of their kindergarten year. The tasks included simple and double seriation, simple classification, two-and three-way classification, class inclusion, conservation of number, and conservation of continuous and discontinuous quantity. At the end of first grade, the 121 children remaining in the project schools were retested on the same set of tasks and were also administered the Metropolitan Achievement Test of reading and mathematics achievement. A modified version of the Metropolitan Readiness Test was administered to 82 of the subjects as part of the district's testing program.

4. Findings

Performance on the Piagetian tasks was consistent with the commonly expected order of difficulty. The easiest task was simple seriation followed in order by double seriation, simple classification, number conservation, conservation of continuous quantity, two-way classification, conservation of discontinuous quantity, three-way classification, and class inclusion. A significant number of children did deviate, however, from this predicted sequence of acquisition.

Most of the correlations between performance on single Piagetian tasks and achievement in reading or mathematics were significant but relatively low. The highest correlations were between the three conservations tasks and mathematics achievement.

To test whether performance on the complete range of concrete operational tasks was a better predictor of achievement than performance on isolated tasks, separate multiple regression equations were constructed for reading and mathematics. When the Metropolitan Readiness Test was entered first into the regression equations, five of the Piagetian tasks still accounted for a significant proportion of the variance. Each of the three subsystems (classification, seriation, and conservation) was represented by one of the predictors in each regression equation.

5. Interpretations

The most plausible hypothesis for the patterns of variability and consistency found in the performance on the Piagetian tests is:

> that there are two or three tiers or levels of performance across which integration and consolidation of the component skills takes place but that within a given tier, or level, individual differences in the order of acquisition are the rule rather than the exception. (p. 720)

Isolated tasks do not provide a good measure of development of concrete operations and performance on one or two tasks tells relatively little about a child's readiness in reading or mathematics. However, a child's general level of operativity as reflected in performance over a range of concrete operations tasks appears to affect significantly a child's achievement in reading and mathematics. The problem remains, however, to identify the specific cognitive demands of basic school tasks and how the limitations of different levels of development affect performance on these tasks.

Abstractor's Comments

This study is one of a number of studies that have consistently found a positive correlation between performance on concrete operational tasks and achievement in mathematics. The study is somewhat more systematic than many earlier studies in the use of a wide range of concrete operational tasks, but the use of global measures of achievement limit the conclusions that can be drawn regarding the utility of Piagetian tasks as readiness measures. The author suggests that the next step should be to identify how performance on specific school tasks is constrained by a child's developmental level. A number of studies have examined this very question. In general, they have found positive correlations between performance on Piagetian tasks and performance on a variety of specific mathematics problem types. However, they also have clearly documented that success on Piagetian tasks is not a preprequisite for success on most school mathematics tasks. The results of these studies and several parallel instructional studies strongly suggest that the basic question of whether Piagetian tasks may be useful as readiness measures should be answered in the negative. In general, this line of investigation does not appear particularly fruitful. For a more detailed review of this literature, see Hiebert and Carpenter (in press).

References

Hiebert, J. and Carpenter, T. P. Piagetian tasks as readiness measures in mathematics instruction: A critical review. Educational Studies in Mathematics, in press.

Falk, Ruma; Falk, Raphael; and Levin, Iris. A POTENTIAL FOR LEARNING PROBABILITY IN YOUNG CHILDREN. Educational Studies in Mathematics 11: 181-204; May 1980.

Abstract and comments prepared for I.M.E. by FRANCES R. CURCIO, St. Francis College, Brooklyn.

1. Purpose

The purpose of this study was to examine the responses of children in different developmental stages and the strategies they used to solve problems requiring a conceptual understanding of probability.

2. Rationale

The authors gave a foundation of previous research supporting the decision-making technique (i.e., after examining two alternatives, selecting the material more likely to yield a success) employed in this study. Although the approach for the previous studies cited was the same, the results were contradictory. The differences were attributed to the use of problems presenting complementary probabilities (e.g., represented by two fractions, one less than ½ and the other greater than ½) in some studies, and a use of problems requiring the comparison of proportions in others. Therefore, the problems constructed for this study included different components of probability in efforts to avoid similar disparities.

3. Research Design and Procedures

The study consisted of two experiments. After recognizing the ability of children (ages 5-11) in Experiment 1, the researchers set up Experiment 2 to examine the responses of children 4-7 years of age. In both experiments, the subjects who voluntarily participated were children from upper-socioeconomic families residing near the Hebrew University of Jerusalem.

Experiment 1. Thirty-six children (20 girls, 16 boys) were tested twice. Different materials were used each time, presenting the children with 22 tasks. The children played individually with two out of three of the following materials:

> (a) Pairs of transparent plastic urns with different compositions of blue and yellow wooden beads in each. (b) Pairs of roulettes of different radii, each with its own distribution into blue and yellow sectors. (c) Pairs of spinning tops of different volumes, likewise subdivided into the same two colors. (p. 184)

The 22 tasks, in which the order of presentation varied, were divided into three levels of difficulty:

> [6] 'Easy' problems, in which one of the two proportions is larger than ½ and the other smaller than ½; [7] 'Medium' problems involving comparisons of ½ with a proportion other than 1; [9] 'Difficult' problems, in which both proportions are either larger or smaller than ½. (p. 187)

Prior to presenting each child with the experimental materials, an introduction to the ideas of a "lottery game" and "uncertainty" were presented by tossing a die three times. Candy was given as a prize if the toss yielded the desired outcomes. Then, one of the materials was demonstrated and each child selected a prize (from among four or five choices) he or she wanted to win. The child was told the "pay-off color" (POC), yellow or blue, and had to choose between two different designs of the same material. The 22 tasks were presented with each type of material.

The same procedure was followed, after a brief intermission, using a different material, also presenting 22 tasks. Children did not have to supply verbal responses and the researchers did not offer verbal reinforcement for correct choices.

Following the lottery tasks, a short interview was conducted with each child. At this time, children were asked to explain their choices by referring to some of the tasks involved in the experiment.

Experiment 2. As a result of Experiment 1, it was found that the roulettes "yielded the shortest sessions on the average" (p. 189), and there were no differences across the three types of materials (p. 189). Therefore, 25 children (15 girls, 10 boys) were individually presented with only roulette materials, with another dimension of probability added (i.e., the number of elements of the "non-pay-off color"). Thirty-two tasks were presented: 10 easy, 10 medium, and 12 difficult. Also, the children in this experiment were instructed to close their eyes (after selecting one of the two roulette wheels) while the roulette dial was

was spinning. Similar to Experiment 1, the order of presentation of the tasks varied.

4. Findings

The results were reported by group responses and by individual responses.

Group Results

1. Using Pearson's coefficient of correlation, results of using pairs of materials were compared. The authors concluded that the three materials (roulettes, tops, and beads) "could be considered to be equivalent instruments for measuring the ability to compare probabilities" (p. 192).

2. The percentages of correct responses were analyzed according to age. In both experiments, a developmental trend was observed; i.e., performance seemed to improve with age.

3. The percentages of correct responses were also analyzed according to task. The authors noted that

> The results . . . suggest that the number of POC elements accounts for most of the variability in the children's responses. The effect of that variable was stronger in Experiment 2, which involved younger subjects . . . (p. 193)

4. "The error of choosing the set with greater number of POC elements is most dominant in the younger ages" (p. 193).

Individual Responses

1. In general, children were not consistent in employing strategies to solve the probability problems. There were a few cases where children's response patterns coincided with a predicted error pattern.

2. Although children's performance might indicate an intuitive understanding of a concept, they might not have adequate verbal ability to express their understanding.

3. The authors analyzed incorrect strategies used by children and presented anecdotal remarks. Some of the children's responses were beyond the authors' imagination (p. 196), whereas others were expected (e.g., selecting the material that had the greater number of POC elements or fewer non-POC elements).

5. Interpretations

(1) After examining children's responses to problems in which ratio and proportions remained constant (although the size and distribution of the POC varied), it was noted that Piaget's principle of conservation also applies to ratio and proportion. The young children were distracted by the number of POCs, when other information (regarding ratio and proportion) should have been integrated to solve the problem. In these cases, comprehension of probability requires the recognition that proportion is invariant "with respect to expansion and cancellation" (p. 197).

(2) Strategies employed in solving problems of probability differ with age; i.e., as children grow older, they become more successful in selecting the set in which they are more likely to win.

(3) Children as young as age 6 or 7 have the potential for learning probability. This finding differs, somewhat, from other research in which concepts of probability were found to be understood at ages 9 to 10. It was mentioned that the differences might have been due to the age groups selected for the sample in the other studies, as well as a possible difference in the socioeconomic status of the subjects.

(4) Incorporating the use of games in classroom activities might contribute to strengthening children's intuitive understanding of probability. The use of these games might enhance children's potential for studying concepts of probability.

(5) Teachers should give children the opportunity to express verbally their conceptual understanding. Appropriate vocabulary could be developed as the need arises while children play games and participate in informal classroom activities.

(6) Children should be exposed to "uncertainty" as early as possible so that they do not develop the misconception that "a correct choice" insures success. This misconception is common in the thinking of many children as well as adults.

Abstractor's Comments

This descriptive, qualitative research study has provided an explanation attempting to resolve some of the apparent disparities that exist in the results of similar studies examining children's understanding of probability. Different components of probability were included, not just complementary probabilities, and not just those requiring comparison of proportions. The three levels of task difficulty were clearly defined. Attempts were also made to design tasks so that children could not get the right answer for the wrong reason, typical n some of the studies cited.

Although correlations between roulettes and beads and between tops and beads (.87 and .91, respectively) are high and can perhaps support using the materials interchangeably, the correlation between tops and roulettes was only .62. This correlation does not fully support the interchangeability of these materials. As a result, this might limit the link made between the results of Experiment 1 and Experiment 2, since Experiment 1 employed the use of the three materials (two per child), and Experiment 2 employed the use of only the roulette material. However, this is no reason to question the conclusions reported.

The comments made about a "conspicuous tendency for performance to improve with age" has to be made and interpreted with caution because this was not a longitudinal study; i.e., children in different developmental stages were not examined over a period of time as they themselves developed.

The example given of a child (Gili, 5 years 11 months) might indicate that young children are not as flexible in accommodating incoming information that might be inconsistent or contrary to what they expect, once they set their mind to it (p. 195). Gili consistently selected the set with the greater number of POC elements. After selecting the correct set, she persisted in choosing the same set, even though the POC was changed and she "did not seem to be disturbed by that fact" (p. 195).

As the authors mentioned, the socioeconomic status of the group of children who were subjects of this study might be a factor in the difference between the results of this study and some of those cited. Children of upper-socioeconomic status seem to have more intellectual stimulation in their home environment.

Finally, this research report has suggested ideas for curriculum development. The need to consider ways of presenting concepts of probability to children (and adolescents) was so important that the National Council of Teachers of Mathematics dedicated its 1981 yearbook to Teaching Statistics and Probability.

Hannafin, Michael J. EFFECTS OF TEACHER AND STUDENT GOAL SETTING AND EVALUATIONS ON MATHEMATICS ACHIEVEMENT AND STUDENT ATTITUDES. Journal of Educational Research 74: 321-326; May/June 1981.

Abstract and comments prepared for I.M.E. by CECIL R. TRUEBLOOD, The Pennsylvania State University.

1. Purpose

The study was conducted to investigate the effects of two types of student regulation and teacher regulation of instruction on student achievement and attitudes.

2. Rationale

Student/teacher regulation was defined as setting weekly learning goals and its relationship to weekly evaluation of goals attained. The goals were computation skills typically taught in grades 4-8. Goal attainment was defined as 100 percent correct responses under mastery learning conditions.

3. Research Design and Procedures

Achievement was measured by performance on a 33-item computation test. Attitudes were measured using a four-item end-of-program survey and a discrete five-point rating scale. Classroom records were also a source of data concerning goal setting and related achievement.

The 2 x 2 factorial design included two levels of goal setting (teachers vs. students) crossed with two levels of evaluation (teachers vs. students). Two sixth-grade classes (N = 48) were randomly assigned to the goal setting conditions. Within each class, students were randomly assigned using a matched-pair technique based upon pretest scores on the school's mathematics program. Students were assigned based upon whether each student evaluated his or her own work or whether the teacher did this evaluation. A statistical analysis indicated there were no differences between classes or among the treatment groups prior to the study.

4. Findings

The authors claim that the results indicated that although teachers set more learning goals for students, students who set their own goals attained proportionately more of them. Students did tend to evaluate their work more favorably than teachers and the work evaluation for teachers and students was higher for students who initially set their own goals. Students reported better goal setting ability when they set their own goals than when teachers set these goals. Self-regulated goal setting and evaluation were significantly related to attitudes but not to mathematics achievement.

5. Interpretations

These findings tend to support those who advocate providing students with more control over instructional goal setting and evaluation of their performance.

Abstractor's Comments

It is not clear whether the "N" for the statistical analyses should be students or classes. The author indicates "classes were randomly assigned." The tables provided suggest that an N of students was used to test the hypotheses. In addition, the Ns provided in Table 1 indicate that there probably were students who did not complete the study. Based upon the original N, there should have been 48 students. Table 1 shows 45. Based upon these and the following limitations, some caution should be exercised in judging the results and the interpretation presented by the author.

Since the study was conducted at only one grade level (grade 6), generalizing the findings to all other grades (especially the primary grades) is questionable. The author does not acknowledge this limitation in his discussion. It should also be noted that the achievement referred to in the study was computation skill. Therefore, generalizing to other areas of the curriculum such as problem solving does not seem warranted.

In general, however, the study does suggest that teachers can delegate some responsibility for setting computation goals to upper grade students without severely affecting their achievement. It also suggests that attitude gains and a feeling of self-regulation could be a benefit from providing students with both goal setting and evaluation experiences.

Hector, Judith H. and Frandsen, Henry. CALCULATOR ALGORITHMS FOR FRACTIONS WITH COMMUNITY COLLEGE STUDENTS. Journal for Research in Mathematics Education 12: 349-355; November 1981.

Abstract and comments prepared for I.M.E. by SUZANNE K. DAMARIN, The Ohio State University.

1. Purpose

The major purpose of the study was to determine whether calculator algorithms offer a viable alternative to traditional computational algorithms for common fractions for community college students who have not previously mastered the algorithms. Attitudes of students using different computational algorithms were also compared.

2. Rationale

In an earlier study by Gaslin, it was found that ninth-grade students who were taught to convert common fractions to decimals and then compute, computed more accurately than comparable students who were taught the traditional algorithms for computing with common fractions.

3. Research Design and Procedures

Seventy-two community college students entering an arithmetic course were assigned to three treatment groups using scores on the Adult Basic Learning Examination Level III subtest of arithmetic computation to obtain matched groups. Each subject was then given three pretests (common fractions understanding and computation subscales of the Stanford Diagnostic Arithmetic Scale). All subjects were given self-paced instruction on whole numbers and an introduction to fractions using the Modumath maerials developed by Hecht and Hecht. The three groups received different instructional treatments for computation with common fractions as follows:

T_1 - Modumath units

T_2 - Modumath units with one session on calculator use and permission to use calculators

T_3 - Instruction on use of calculators followed by instruction on operations on common fractions by first converting them to decimals

Following instruction, the three measures were readministered to all students remaining in the course. Data were analyzed using multivariate analysis of variance (16 subjects per treatment group), followed by a discriminant function analysis.

4. Findings

There were no differences between treatment groups, nor any interaction between treatment and examination. There was, however, an effect for examination ($p = .0001$); discriminant function analysis showed that this effect was due to changes by all treatment groups on the computation subtest of the SDAS.

5. Interpretations [added by editor]

The significant pretest to posttest gain was an indication that in all three treatments students were able to learn fraction computation algorithms. The calculator algorithms can serve as an effective alternative instructional strategy where computational skill is a goal of instruction.

Though the instructional materials stressed the noncommutativity of subtraction and division and the importance of the order of entering numbers in the calculator, this aspect of calculator algorithms is difficult for students.

Abstractor's Comments

This study is poorly conceptualized and, therefore, adds nothing to our understanding of the teaching or learning of fraction operations in community colleges. Even if the experimental group had far surpassed the other groups in posttest performance, the meaning of the results would be unclear. Since the results are "no significant differences" between groups, we do not need to grapple with the issue of interpretations.

A good study of this topic would need to include attention to students' beliefs concerning "equivalence" of fractions and decimals, as well as better rationalization of teaching and scoring procedures.

Hirsch, Christian R. AN EXPLORATORY STUDY OF THE EFFECTIVENESS OF A 'DIDACTICAL SHADOW' SEMINAR IN ABSTRACT ALGEBRA. School Science and Mathematics 81: 459-466; October 1981.

Abstract and comments prepared for I.M.E. by KENNETH A. RETZER, Illinois State University, Normal.

1. Purpose

The study explored the feasibility of implementing a "didactical shadow" seminar in abstract algebra and provided preliminary empirical data regarding the efficacy of such a seminar. Effects of the seminar on the understanding of concepts and principles of abstract algebra and understanding the algebra of real numbers by prospective secondary mathematics teachers (PSMTs) were examined, as well as the seminar's effects on student attitudes toward mathematics.

2. Rationale

The paucity of curriculum development and related research with respect to the mathematical preparation of PSMTs was cited. "Shadow" seminars were suggested in the Snowmass Conference (Springer, 1973).

3. Research Design and Procedures

The study employed a pretest-posttest control group design (Campbell and Stanley, 1967) contrasting test scores of a treatment group of nine PSMTs enrolled in a four-semester hour undergraduate abstract algebra course at Western Michigan University against a control group of seven PSMTs enrolled in the subsequent semester.

Content was pretested with Algebra Inventory, Form A (Begle, 1972) and attitudes were tested by the Aiken-Dreger Revised Mathematics Attitude Scale augmented by scales measuring attitudes towards mathematics as a process and the place of mathematics in society from the International Study of Achievement in Mathematics (Husén, 1967). In the posttest the content was assessed by Algebra Inventory, Form B and Abstract Algebra Inventory, Form C (Begle, 1972) and the attitude scales were readministered. Means, standard deviations, and mean differences were reported and analyzed statistically.

Materials gathered and developed by the researcher are listed in a table in the research report. An attempt was made to sequence the topics so that their temporal treatment corresponded to the use of the groups-rings-fields progression of the abstract algebra course.

4. Findings

Using a correlated t-test, the only significant difference found was in the improvement in understanding of algebra of the real number system in the case of the experimental group. There was also improvement in the content understanding of the control group. In no instance was there a decline in the attitude scores of either group.

No significant differences in understanding were found between the experimental and control groups as t-tests for independent samples were applied to the means of the pretest-posttest difference scores. Neither was there a significant difference found between the two groups on the abstract algebra inventory by a t-test applied to mean scores.

The researcher also examined the distribution of letter grades of the two groups using a chi-square test with Yates' Correlation for Continuity and found no differences in the proportion of subjects receiving "A" grades or "A" and "B" grades. A questionnaire permitting open-ended answers was administered and responses were generally favorable. Sample comments were quoted.

5. Interpretations

In discussing the study, the researcher felt the most important outcome was that a shadow seminar is feasible and can be reasonably implemented within the structure of a university secondary mathematics teacher education program. He concluded that the question of the efficacy of the "shadowing" concept remains open and encouraged further research which might include the seminar's effects on specific teacher variables or the ability of participants to promote pupil learning as dependent variables.

The limitations of having a small number of subjects were noted. The fact that the size of the abstract algebra classes may have promoted greater than usual motivation and understanding was also pointed out.

A small-group method of instruction permitted students to be actively engaged in proving theorems, working with examples and counterexamples, and formulating conjectures which may have added a certain concreteness to this abstract algebra course. Implementation of a shadow course was also seen to relieve the severe time constraints on secondary mathematics education methods courses.

Abstractor's Comments

Both secondary mathematics teachers and teacher educators should find Hirsch's research and development interesting because it confronts the question of providing the best possible content preparation of secondary mathematics teachers and the common PSMT perception of lack of relevance of abstract algebra content to secondary mathematics.

Scarcity of significant results should not be regarded as discouraging because those engaged in research contrasting methods frequently get such findings. The insignificance of differences does provide evidence to allay fears that implementation of a shadow seminar might affect attitudes toward mathematics negatively or contribute to a less adequate comprehension of the content.

Curriculum innovators would be undaunted by the results, for they recognize that most curricular changes are not supported by research findings as to their effectiveness. One mathematics educator observed that two basic tasks of humans were to hold values and make decisions (Brown, 1982), and the content and sequencing of courses in our curriculum seem overwhelmingly to result from our values and our decisions rather than efficacy studies. The existence of such shadow seminars at Southern Illinois University and the development of them at the University of Minnesota attest to interest in them in the mathematics education community. Mathematics educators may be as interested in discussing with the researcher the intricacies of getting such a course proposal through curriculum committees and discussing the content of the seminars themselves as they are in the research report.

It would seem that an inservice teacher who could not see the significance of his or her abstract algebra course could seek and read the articles in Hirsch's list of references upon which the various shadow seminar sessions were based and independently explore those connections. A teacher educator could use the same list together with copies of material Hirsch has developed as suggested readings for current mathematics education courses or as content with which to assemble a shadow seminar of his or her own.

Thus, the development of the shadow course and its underlying sources can be of value to both teachers and teacher educators, and the research report will enable those interested in pursuing efficacy studies to refine both the treatment and the dependent variables in order to extricate solid evidence of benefits which their sense of values tells them are there and yet undetected.

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Houlihan, Dorothy M. and Ginsburg, Herbert P. THE ADDITION METHODS OF FIRST- AND SECOND-GRADE CHILDREN. Journal for Research in Mathematics Education 12: 95-106; March 1981.

Abstract prepared for I.M.E. by CHARLES DE FLANDRE,
University of Quebec, Montreal.
Comments prepared for I.M.E. by CHARLES DE FLANDRE and KAREN FUSON,
Northwestern University.

1. Purpose

The purpose of this study was to analyze the procedures used by first- and second-grade children in solving addition problems and to extend previous work on this subject.

2. Rationale

The studies of Hebbeler (1978), Posner (1978), Ginsburg (1978), and Zaslavsky (1974) have shown that techniques for dealing with addition develop before the onset of schooling or without the benefit of schooling in non-literate cultures. These informal techniques then have effects on how children learn to solve addition problems. The "few studies" of Groen and Parkman (1972), Suppes and Groen (1967), and Russell (1977) which deal with addition strategies of children in the early grades indicate that early counting strategies developed before the onset of schooling play an important role in determining what procedures are used in school and that the methods children use are not necessarily the ones presented through formal instruction. This study attempted to extend the previous research on early addition strategies.

3. Research Design and Procedures

The subjects were 56 children, 25 first graders and 31 second graders, from a parochial school in Ithaca, New York. Most of the children were from low- and middle-income families and they came from the same two classrooms.

The clinical interview technique was used in order to obtain detailed information about the strategies which the children employed to solve six addition problems. The problems were presented orally to half the subjects and in writing to the other half. On only the second, fourth, and sixth problems each subject was to describe the solution procedures used. Within each grade equal numbers of boys and girls were placed in each of the oral and written conditions.

The three interview problems varied in magnitude; one contained two single-digit addend (S-S) problems; another, one single- and one double-digit addend (S-D) problem; and the third, two double-digit addend (D-D) problems without regrouping.

Children's responses to the clinical interviews were recorded verbatim and then coded into the following categories:

<u>Non-counting Methods</u>

1. Direct memory (3 + 4 is 7).
2. Indirect memory (5 + 7 ⇒ 5 + 5 + 2 ⇒ 10 + 2 ⇒ 12).
3. Place value (23 + 16 ⇒ adds 3 and 6 and adds 2 and 1, puts the sums in the appropriate columns).

<u>Counting Methods</u>

4. Counting from 1 with concrete aids (fingers, marks on a piece of paper, poker chips) (2 + 3 ⇒ 1, 2,3,4,5).
5. Counting from 1 without concrete aids (counting out loud without the use of fingers or poker chips).
6. Counting on with concrete aids (6 + 4 ⇒ child either counts from 4 or 6).
7. Counting on without concrete aids.
8. Indirect memory and counting (8 + 3 ⇒ 3 + 3 + 5 ⇒ 6 is memorized, then counting begins from 7 to 11).
9. Indeterminate counting (child uses some kind of counting method but the coder cannot categorize it more specifically).
10. Inappropriate method (no answer, uses subtraction or changes number).
11. Indeterminate (child gives an answer but cannot describe how she or he obtained it).

To determine the reliability of this categorization, an independent judge was asked to categorize a randomly selected sample of the descriptions by 10 children of their addition methods on the three interview problems. There was 95% agreement between the categorization of the interviewer and those of the independent judge.

Before analyzing the data in depth, the Fisher Exact Probability

Test (F.E.P.T.) was used to see if there were any significant differences between two problem sets and between the two methods of presentation: written and oral.

4. Findings

For the preliminary analysis the F.E.P.T. produced non-significant differences between each problem set relative to: a) the number of children answering correctly on each problem size (S-S, S-D, D-D) for both grades, and b) the number of children using appropriate methods on each problem size. Because of one exception which the F.E.P.T. produced concerning the non-significant differences between the Oral and Written Presentation formats, the data for the two problem sets and the presentation formats were pooled for further analysis.

The results of the sign test used for the matched data (S-S was S-D and S-D was D-D) indicated that in both grades problem size had a greater effect on the children's ability to solve a problem correctly than on the ability to apply an appropriate method to a problem. Thus they were "able to apply approoriate methods to problems of larger size than they were experienced", but were unsuccessful in their attempts.

The data which the authors compiled in a table of distribution of strategies on each problem size indicated that counting methods to solve each of the problems were used by first graders and that both counting and non-counting methods were used by second graders.

5. Interpretations

According to the authors, the results of the study coincide with those of other researchers:

a) Although the method of this study was different from that of Groen's (1972) and Suppes (1967), it confirmed that fact that the most frequently used methods by first graders are counting on from the larger addend and counting from one starting with the first addend.

b) Russell's study (1977) with third graders and this study both indicate that second and third graders use both counting and non-counting methods, and when counting is used it involves counting-on procedures. Apparently with time and instruction methods become more economical.

c) Second graders who used non-counting methods employed the same strategies of addition by place value as the third graders in Russell's study.

d) The authors concur with Groen and Resnick (1977) that since children assimilate what is taught into what they already know, then place value methods are "invented strategies"; and that as problems become more complex they tend to create "more and more non-counting-based invented strategies".

e) Again as with Russell's study (1977), the results of this study show that children vary their strategies according to the level of the children.

A particular result of this study which is different from previous studies is that first-grade children found the larger addition problems easier to attempt when presented orally than when presented in writing. The interpretation the authors give of this fact is that first graders are more used to hearing double-digit numbers than they are to reading these numbers. This difference may be eliminated by the second grade since the data show there were no differences between second-grade oral and written groups.

The authors indicate that their data have three implications for education:

a) Second graders can independently apply to addition problems their knowledge of place value.

b) Since the results show that not all children in a class use the same addition methods which are either taught in class or invented by the children, educators could attempt in class to explain how the different methods are related. The educator could also encourage children to invent different methods and then compare them to evaluate their accuracy and efficiency.

c) Since children apply ~~appropriate methods~~ but do not always use them accurately, an educator, when making evaluation, could also examine the methods used rather than just measure correct responses.

Abstractor's Comments (1)

It seems that this study was adequately designed and conducted for a master's level thesis. It confirms results of previous studies on the same subject as well as what is probably apparent to any teacher who is observant of the behavior of his or her children. There are some questions, however, which need to be discussed.

The published report

Should this level of study be published in the Journal for Research in Mathematics Education? This question is not to imply that a master's level research should not be published in this journal. But since this study seems to have particular implications for education on a pedagogical level, might not the results be best communicated to teachers in another journal?

It might have been helpful to educators who would like to do a related study to have some suggestion included in the report as to possible further questions to investigate.

The research

a) The author's statement, "In both grades children were able to apply appropriate methods of a larger size than they were experienced with, but were often unsuccessful in their attempts," could have been illustrated with some examples. What are the author's criteria for evaluating an appropriate method? If a child added on the D-D level from left to right, would that procedure be considered inappropriate? What is meant by "unsuccessful in their attempts?" Was an analysis made of the responses of the children in the group on the 65% of first graders who did not use appropriate methods?

b) The authors indicate that each child was given six addition problems, but it is not clear why the six had to be given since only three were to be used during the interviews.

c) Since the presentations were given in the written form a + b, would there have been any significant differences between the oral presentations and the written form $\begin{array}{r} a \\ +b \\ \hline \end{array}$ and between the two written forms a + b and $\begin{array}{r} a \\ +b \end{array}$? The present writer has observed that in many classrooms where only the format a + b is given to them, children have difficulty with the format $\begin{array}{r} a \\ \underline{+b} \end{array}$ because for them the later symbolization has no meaning and the first format has been learned by a stimulus-response teaching approach.

For the D-D examples, would there be any significant differences if the written or oral presentations had been of the form 23 + 12, where the first number is larger than the second?

d) The term "problem" may be misleading in the context of this study. The stimuli given to the children are examples or exercises. If the stimulus has meaning for the child, he or she will apply a procedure which can be recorded by the interviewer. But if the stimulus has no meaning for the child, but he or she has learned a procedure and applies it correctly to obtain a correct result, what significant conclusion can be drawn? Let us examine the case 23 + 16. Is this a problem by definition for the child? It depends: if the child has had no experience with place value, what meaning can it have for him or her? The child has no existing schema which permits him or her to assimilate the stimulus. It therefore cannot be considered a problem. However, if the child has had meaningful experiences with place value, if 23 and 16 have a meaning for the child, and if the child has not been drilled on addition of two digit numbers, then 23 + 16 may be considered a problem for the child.

e) If the children had not been previously given additions in class of the types (5 + 3 + 4 + 2) or (23 + 14 + 12) and if these types had been used in the interview, would the strategies used have been different?

f) Because of the complexity of the concept of place value system of numeration, it is questionable whether the children who made a correct response to 23 + 16 were actually using a place value strategy. It would seem that in order to verify if a child really uses this strategy the child would have to be able to successfully find the results to 432 + 527 or to $\begin{array}{r} 23 \\ \underline{+16} \end{array}$

g) It is stated in the report that the textbook for <u>both grades</u> presented place value concepts, particularly as they apply to the writing of numerals. In the table of data it can be noted that while 77.5% of the second-graders used appropriate strategies for D-D, only 35% of the first-graders used appropriate methods for the same level exercises. It would seem then that prehaps instruction had an influence on the results. To what degree are the appropriate strategies related to the instruction? The present writer has observed that children who have developed an understanding of the place value concept in the first grade (that is, children are able to describe and illustrate the meaning of symbols 3762 without naming the columns) develop appropriate strategies for adding 3762 + 4879 in the second grade <u>without being taught a particular procedure</u>.

Example of	3762
a particular	+4879
strategy	7000
observed:	1500
	130
	11
	8641

In brief, this study confirmed findings of previous studies on the subject, but unfortunately does not give new significant insights.

Charles de Flandre

<u>Abstractor's Comments (2)</u>

This article is one of several concurrent but independent efforts that examined the solution procedures which young elementary school children use in addition and subtraction problems. These papers are now appearing in print, and they give us a much more complete and richer picture of the capabilities which these children possess (even aside from instruction). Some other such papers are Carpenter, Hiebert, and Moser (in the January 1981 issue of the <u>Journal for Research in Mathematics Education</u>), and several papers (by Carpenter and Moser; by Steffe, Thompson, and Richards; by Fuson; and, for younger children, by Gelman and Starkey) which will appear in the book <u>Addition and Subtraction: A Developmental Perspective</u>, edited by Romgerg, Carpenter, and Moser (to be published by Lawrence Erlbaum Associates). All except the last paper deal with the counting-on procedure.

The Houlihan and Ginsburg paper contributes to our knowledge in several ways. First, the problems given go beyond the usual limits of addition problems taught to first graders (sums below ten) to include double-digit numbers. Second, the formal teaching content which the children had experienced was assessed at least somewhat so that conclusions about children's inventions could be made. Third, performance was contrasted when problems were given orally and in written form. Finally, specific solution procedures rather than mere correct or incorrectness of response were the main focus of interest. In general, the conclusions which the authors draw seem warranted by their data. These are outlined above in the extended abstract.

A few minor questions might be raised about specific results and interpretations of the study. Two of these concern issues about which the interview method seemingly would have enabled some resolution. First, the authors propose a possible interpretation of the finding that significantly more first graders used correct solution procedures for the single-double digit problems when they were presented in oral than in written form. This interpretation is that first graders have relatively more difficulty in recognizing double-digit numbers by reading than by listening. If in fact this was the case, it would seem that the interview method would have permitted the authors to observe the nature of any difficulties children had in recognizing written forms. For example, did children reverse the digits of the double-digit number or were they simply unable to verbalize the problem at all? If the latter, does this indicate that children's meanings of double-digit number words are first primarily auditory ones, and that these auditory forms of the written numerals must be accessed for a solution to be reached? This would seem to be a significant finding and could be indicated simply by the number of children in the written condition who in fact could not give the oral form of the two-digit number.

A second issue is that a sizable number of children counted-on without objects in the single-digit double-digit and double-digit double-digit conditions. How did these children keep track of how many they were counting-on (they were counting-on at least 7, 13, or 14 for the smaller numbers in those problems)? Again, the interview method would seem to have permitted some indication of how these children were doing this successfully.

A final minor point is that the authors conclude in their abstract and article that "In general, second grade children efficiently adjusted their strategies according to the magnitude of the problem's addends." The words "efficiently adjust" are probably too strong here. The fact is that different strategies are possible for problems with addends of different sizes (e.g., one cannot use a place value solution on a single-digit problem and one does not memorize solution facts for double-digit problems). The data thus indicate that first and second graders in fact have these different strategies available for addends of different sizes, and they will use different strategies for different sizes. Because the study only assessed the first method each child used, and not all of the methods which that child <u>could</u> have used, we do not really know if each child used at each level the most efficient strategy available to him or her. Individual children in fact may not have made the most efficient selection of the strategies available to each.

Karen C. Fuson

Karmos, Joseph S.; Scheer, Janet; Miller, Ann; and Bardo, Harold. THE RELATIONSHIP OF MATH ACHIEVEMENT TO IMPULSIVITY IN MATHEMATICALLY DEFICIENT ELEMENTARY SCHOOL STUDENTS. School Science and Mathematics 81: 685-688; December 1981.

Abstract and comments prepared for I.M.E. by THOMAS C. GIBNEY, The University of Toledo.

1. Purpose

To investigate the relationship between impulsivity and learning mathematics.

2. Rationale

Impulsivity involves the degree to which a student reflects on the validity of a hypothesis or answer to a problem that contains response uncertainty. Past research suggests that impulsivity may hinder a student's learning of mathematics. This study related to previous findings between impulsivity and computation by Jon Englehardt and Albert Rebhun. Other related studies by S. B. Messer, A. Schmebel, R. M. Yando, and J. Kagan were surveyed in the article.

3. Research Design and Procedures

Fifty-five elementary students of age 12 or less were the subjects. There were 25 girls and 30 boys in the study, all clients of the Southern Illinois University Diagnostic Mathematics Clinic, the Mathematics Learning Clinic at Arizona State University, or the Arithmetic Center at the University of Maryland.

All instruments were administered by a clinician on an individualized basis at the student's first session at the respective clinics. Impulsivity was measured by the number of errors on the Matching Familiar Figures Test. The KeyMath Diagnostic Arithmetic Test (1976) was used to determine the extent of the mathematics deficiencies and yielded mathematics scores in the 14 different content, operations, and application areas of the test.

4. Findings

Moderately strong relationships were found between impulsivity scores and scores for each of the 14 KeyMath areas. For seven of the content, areas, the absolute value of the correlation was 0.45 or greater. Impulsivity, therefore, accounted for at least 20% of the variability in mathematics scores for one-half of the mathematics areas. The correlations ranged from -0.31 to -0.48 with seven scores in the .30s and seven in the .40s. Of the 14 KeyMath areas investigated, Mental Computation had the strongest relationship with impulsivity ($r = -0.48$), while multiplication had the weakest relationship ($r = -0.31$).

5. Interpretations

Students indentified as mathematically deficient obtained moderately strong correlations between inpulsivity and 14 areas of mathematics. These correlations suggest that elementary school students who have mathematics deficiencies might benefit from specific training to reduce impulsivity in certain mathematics areas, particularly mental computation.

Abstractor's Comments

It is particularly important that the signs of all r's were negative and that they were quite consistent in magnitude. These facts should have been discussed in more detail in the article.

With $n = 55$, it takes an r of about 0.27 for a two-tailed test, to obtain significance at $\alpha = 0.05$. If confidence intervals were built for the r's, some would be close to spanning zero.

Just because $r = -0.48$, it does not necessarily follow that impulsivity causes the variability in the mathematics scores. Correlation does not necessarily imply cause and effect.

The researchers have compiled evidence to support the benefit of training elementary students to be less impulsive about computation in mathematics. Instruction designed to test this evidence appears appropriate.

McClinton, Sandra L. VERBAL PROBLEM SOLVING IN YOUNG CHILDREN. Journal of Educational Psychology 73: 437-443; June 1981.

Abstract and comments prepared for I.M.E. by JAMES H. VANCE, University of Victoria, Victoria, British Columbia.

1. Purpose

The purpose of the research was to study the ability of children at three age levels to deal with class inclusion problems presented in three sensory modalities: verbal, visual, and kinesthetic. Both response accuracy and correctness of reason were investigated.

2. Rationale

One effect of Piaget's theory of cognitive development on early childhood education practice has been an emphasis on learning by doing and manipulation of materials. While acknowledging that children prefer and benefit from active participation based on visual and kinesthetic modalities, the investigator questions the implication that children are not able to process verbally and suggests that this emphasis on these modalities may have "obscured the child's verbal capabilities" (p. 437).

Piaget's class inclusion problems were chosen for the study because they can be presented visually, kinesthetically, and in a purely verbal form, and also because the literature suggests that the solution to these problems is related to the mode of presentation. Researchers have also attempted to find explanations for children's incorrect responses to class inclusion problems. The misinterpretation hypothesis holds that children translate the question into a comparison the two subclasses, giving the larger of the subclasses as their answer.

3. Research Design and Procedures

The subjects were 72 children attending five schools in middle-class neighborhoods of a suburban city. There were equal numbers of boys and girls at each of three age levels: 4-, 6- and 8-year-olds.

Two problems were given to each subject under each of three conditions:

(a) The verbal presentation. No materials were used. For one problem, the experimenter would say: "Let's pretend I have a box of shapes. In the box is a little pile of squares and a big pile of circles. In the box are there more shapes or more circles? Why?"

(b) The visual presentation. A color photograph was shown to the subject. For the shape problem, the photograph showed a small pile of circles and a larger pile of squares. The experimenter would say: "These are circles and these are squares (pointing). In the picture, are there more squares or more shapes? Why?"

(c) The kinesthetic presentation. For the shape problem, a box containing circles and squares (more circles) mixed together was placed on the table. The experimenter would say: "Here are some shapes. Watch me", and sort the shapes into two piles. Then the experimenter would replace the shapes in the box and ask the child to sort them. After correcting the pile where necessary, the experimenter would say: "These are circles; these are squares. Are there more circles or more shapes? Why?"

The other problem involved crayons, pencils, and things that write. Within each age group, subjects of each sex were randomly assigned to one of the six possible condition orders, and within each condition order to one of the two problem orders. Problem order assignments also determined which one of two examiners (one male and one female) was used.

To correct for a possible tendency for subjects to repeat previous answers or select the last alternative, the majority subclass was reversed for each condition and between problems within conditions, and the order in which the subclass and the superordinate class were mentioned was alternated. Thus, if a child was given the verbal condition first and the visual condition second, the first three questions might be: "Are there more shapes or more circles? Are there more crayons or more things that write? Are there more squares or more shapes?"

A mixed model experimental design was used. Within-subject factors were conditions and problems; between-subject factors were sex, age, examiner, and order.

4. Findings

An analysis of variance computed to examine response accuracy revealed the following significant effects ($p < .05$): Condition ($p < .025$), Age X Order interaction ($p < .05$), and Age X Condition interaction ($p < .005$).

Using the Newman-Keuls multiple comparison procedure on the three conditions means, it was found that the subjects in the verbal condition gave significantly more correct responses than those in either the kinesthetic or the visual condition. There was no significant difference between the latter two conditions.

An inspection of the Age X Order graph suggested that the 8-year-olds outperformed the younger children in four of the six condition orders.

Table 1 (p. 440) summarizes the results of the Age X Condition interaction. The Newman-Keuls procedure indicated that the 4-year-olds did significantly better under the verbal condition than under either the kinesthetic ($p < .001$) or visual ($p < .01$) conditions. No significant condition effect was found for the two other age groups.

Table 1

Percentage of Correct Responses by Age and Condition

Age (years)	Verbal	Visual	Kinesthetic	Total
4	40	19	13	24
6	17	15	8	13
8	42	42	44	42

Reasons given by subjects for their responses were of three types: don't know; reason based on a comparison between the two subclasses; and correct reason. The "don't know" reason was given more often by the 4-year-olds than by the older children, and occurred more frequently in the verbal condition than in the other two conditions. The 8-year-olds gave more correct responses than the 4- and 6-year-olds; there was no significant difference in this regard between the two younger groups.

The ability to supply correct reasons for correct answers increased with age under all conditions. The majority of reasons given across grade levels was of the type that indicated children were comparing the two subclasses.

5. Interpretations

Since the 4-year-olds performed better in the verbal condition than in the visual or kinesthetic conditions, support is given to the contention that certain problems may be made more difficult for young children by insisting on visual or kinesthetic presentation. The verbal presentation may allow the young child to focus attention on the problem itself, without being distracted by pictures or objects. Emphasis on touch and sight in early childhood may place the child at a disadvantage in coping with verbal information. There should be equal emphasis on activities which involve the verbal modality without visual or concrete props.

Abstractor's Comments

This is a worthwhile and well-designed study; the results contribute to the literature on both class inclusion problems and concrete-pictorial-verbal modes of learning. However, the investigator's interpretation of the data and the implications she draws for instruction are open to debate. The presentation of the data in the report allows the reader to formulate alternative interpretations of the results.

Consider again Table 1. Looking only at the 4-year-old results, one might be led to conclude, as did the investigator, that children can solve problems more easily when they are presented in a purely verbal way, and that one explanation for this might be that pictures and objects are distractions rather than helpful aids.

My experience with young children makes it difficult for me to believe that 40% of the 4-year-olds could correctly assimilate the information and relationships in these problems (e.g., shape, little, square, big, circle, more) in the verbal presentation. Data supplied in another table in the report increase my doubts: under the verbal condition, no correct reasons were given by 4-year-olds for correct answers, and 79% of the reasons given by these children were of the "I don't know" type. On the other hand, in the visual and kinesthetic conditions, 11% and 17% respectively of the correct answers were accompanied by correct reasons. Across conditions, the percentages of correct reasons for correct answers increases with age (to 35% at age 8). Across ages this ratio increases from 32% in the verbal condition, to 58% and 74% in the visual and kinesthetic conditions. These figures clearly do not support the claim that the verbal modality is superior.

Then how does one explain the 40%? First, it should be remembered that the subjects were not requ' ed to "solve" the problems or supply answers; they were simply asked to select one of two alternatives provided for each question. In a normal problem-solving situation then, about 50% of the answers would be correct by chance. Class inclusion problems, however, are cognitive development tasks and it is expected that children who are not at a particular stage will select the wrong alternative, not because they are guessing, but because of the way they perceive the situation. Now if at the verbal level the 4-year-olds could not understand the problem, their answers would reflect random guessing. In other words, perhaps the younger children didn't comprehend enough to be misled. When the problems were presented with pictures and objects, the percentage of correct answers decreased because some of the children began to understand the situation well enough to misinterpret the question. The ratio of "correct" answers for "wrong" questions is greatest in the kinesthetic and visual presentations, and only chance in the verbal presentation.

The investigator states: "For the 8-year-olds, the mode in which the problem was presented was not so crucial, indicating that the older child is not so highly influenced by distracting visual and kinesthetic cues" (p. 443). An opposite explanation would be that at age 8 the child is developmentally more able to function at the verbal level without the direct aid of pictures or objects, with which he or she has had previous experience.

With respect to the investigator's concern that preschoolers are not given sufficient opportunities to learn and solve problems at a purely verbal level, it is not clear what specific changes from current practice might be suggested. Children are encouraged to listen (to stories and instructions), to imagine situations, and to speak and express their ideas. Activities in visual and kinesthetic settings are usually accompanied by verbal directions or questions. Number and geometrical ideas certainly must be taught initially with reference to objects and pictures. But teachers can ask questions and pose problems at a verbal level about concepts and terms with which children are already familiar from previous experience.

In conclusion, while the investigator's concern about the ability of young children to learn at a verbal level may be justified, the results of this study on class inclusion problems do not support the conclusion that verbal presentations are superior to pictorial and kinesthetic presentations in early childhood.

Nibbelink, William H. COMPARISON OF VERTICAL AND HORIZONTAL FORMS FOR OPEN SENTENCES RELATIVE TO PERFORMANCE BY FIRST GRADERS, SOME SUGGESTIONS. School Science and Mathematics 81: 613-619; November 1981.

Abstract and comments prepared for I.M.E. by DOUGLAS A. GROUWS, University of Missouri, Columbia.

1. Purpose

The purpose of this study was to (1) determine if first-grade pupils' solving performance on open sentences presented in vertical form differed from their performance on similar sentences presented in horizontal form and, if so, 2) ascertain whether the differences were due to different perceptual skills being used in processing each form.

2. Rationale

It is known that young children discriminate between figures initially by vertically scanning for differences and then by horizontally scanning. The accuracy of the horizontal scanning skill develops more slowly than the vertical scanning skill and may thus be associated with reversal errors in processing horizontal form sentences. Similar errors may not occur on vertical form sentences, since the vertical scanning skill develops earlier, and thus solving performance may be better on vertical forms than on horizontal forms.

3. Research Design and Procedures

The general equation $\underline{a} \circ \underline{b} = \underline{c}$ was considered. Two operations (addition and subtraction), two modes of presentation (vertical form and horizontal form), and three placeholder positions ($\underline{a}$, $\underline{b}$, and $\underline{c}$) were used to form 12 item types. Four items of each type were constructed using basic fact combinations to generate each 48-item test. Test forms were balanced to insure that basic fact difficulty did not operate differentially within the primary factor of interest: mode of presentation. Thus, if $8 + \square = 13$ appeared on one test, then $\begin{array}{r} 8 \\ +\square \\ \hline 13 \end{array}$ was on a different test.

The tests were administered to 40 first graders (8 randomly selected from each of five first-grade classrooms). The five sample classrooms had participated in a curriculum development project "aimed at curbing children's tendencies to do-as-the-sign-says" (p. 615) when solving sentences like 3 + /__/ = 7; that is, to avoid always combining the two given numbers using the given operation regardless of the nature of the open sentence.

4. Findings

Pupils correctly solved significantly ($p < .01$) more vertical form sentences (74%) than horizontal sentences (66%). Omitting straight basic fact sentences, where there were no practical differences in performance, 70% of the vertical form (V) sentences were correctly solved as opposed to 58% of the horizontal form (H) sentences.

The tests were rescored, "counting as correct answers which could be the result of either complete reversing or filtering and reversing" (p. 616). By rescoring the tests, scores on the H sentences improved and there was less than one percent difference in performance between H and V sentences.

5. Interpretations

The form (H or V) did not affect performance on straight computation sentences, perhaps because of substantial previous work with such types of sentences exclusively and/or the close proximity of the numerals to the operation sign.

The largest differences between H and V were on sentences where the placeholder was in the initial position (i.e., the ~~a~~ position). These differences disappear when reversals are taken into account (i.e., scored as correct), thus suggesting that pupils switch from one method of attack to another much more readily with H than V. Surprisingly, results on sentences of type 3 (i.e., a - /__/ = c) showed higher scores for H than V, which may mean that pupils treat it as a special case.

"The horizontal form for open sentences should be avoided in grades one and two because the perceptual skills required by that form are not well established and because open sentences offer a poor vehicle for teaching such skills" (p. 619). Open sentences in vertical form, however, can be both meaningful and enjoyable for younger pupils, and certainly sentences in this form can be used as enrichment for academically talented pupils.

Abstractor's Comments

This is an interesting investigation and as such raises a number of issues and questions. My first comment is that it is unfortunate that none of the substantial literature related to open sentences is presented or reviewed. In fact, a study reported at AERA in 1972 dealing with performance on V and H formats in workbooks was not even mentioned.

The author does take account of instruction in discussing the results of the study by pointing out that the instructional program with its emphasis on lowering the incidence of do-as-the-sign-says strategies provides an advantage to detecting left-right reversal errors. In my view the nature of the instruction, the emphasis or lack of emphasis on certain sentence types or forms, and other instructional considerations can greatly affect solving performance. The author acknowledges this to a limited extent when he explains the lack of differences in H and V direct solution sentences. He seems inclined, however, to minimize the importance of instructional program in explaining H and V differences and relies much more on inherent characteristics of the sentences and their possible link to perceptual problems. At a minimum, it would have been useful to have a careful description of the instruction (the context of the instruction--how placeholders were introduced--was clear) so that readers might at least speculate on their own about instructional effects or bias.

The author does a good job of analyzing the data by scoring and rescoring the data on the basis of correct answers. Further analysis or descriptions of incorrect responses would have been interesting. In fact, an attempt to determine whether students consistently used a particular strategy (e.g., do-as-the-sign-says) or tended to be haphazard would have been useful information. In fact, the study cries out for some observational data or interview data to supplement the analyses of the test data. Perhaps, in fairness, it was not feasible to collect such data due to this study being but a part of a larger project. Clinical work does seem to be a logical extension or follow-up to this investigation.

Finally, the author in my opinion jumps overboard in his final paragraph of suggestions, after having been particularly careful and cautious in all the previous discussion. I think he goes beyond the level of prudence when, based on a single study, he concludes that "horizontal form for open sentences should be avoided in grades one and two" (p. 619). This seems particularly blatant when one recalls that the first-grade pupils in this study scored 66% correct on the horizontal open sentences overall and 82% correct on the straight computation horizontal sentences.

Peterson, Penelope L.; Janicki, Terrence C.; and Swing, Susan R. ABILITY X TREATMENT INTERACTION EFFECTS ON CHILDREN'S LEARNING IN LARGE-GROUP AND SMALL-GROUP APPROACHES. American Educational Research Journal 18: 453-473; Winter 1981.

Abstract and comments prepared for I.M.E. by BILLIE EARL SPARKS, University of Wisconsin, Eau Claire.

1. Purpose

This was an aptitude treatment interaction study of fourth and fifth graders learning geometry. Specifically, will curvilinear regression show an interaction between treatment (small group versus large group) and aptitude? Also, the study investigated the effect on achievement of matching preferred learning style with instructional approach.

2. Rationale

Several studies of small-group learning have found that while there may be no significant main effect for treatment (small group versus large group); there is an interaction between aptitude and treatment. Several of the researchers who have found such aptitude-treatment interactions have hypothesized that in small groups high-ability students give explanations and learn from that, low-ability students receive explanations and learn from that, while middle-ability students neither give nor receive explanations. This study was an attempt to replicate the existence of such an aptitude-treatment interaction and to explain it through observation of the group process present.

The authors had conducted a previous study in which students performed worse when taught in the approach they had initially preferred. Since this is contrary to common belief, the study also sought to investigate this finding further.

3. Research Design and Procedures

Ninety-three fourth and fifth graders from a single elementary school in Stoughton, Wisconsin, were the subjects utilized for the study. They were taught by two experienced teachers, each teaching approximately one-fourth of the students by each of the treatment procedures.

Prior to the treatment, each student was assessed by a mathematics achievement test (STEP, Mathematics Basic Concepts), an IQ measure (Raven's Progressive Matrices), a locus of control measure (Academic Achievement Accountability), and three experimenter-designed instruments which assessed attitude toward mathematics, preference for learning in small groups, and under what procedure the student felt he or she would learn best.

A stratified random assignment was then made to teacher and treatment so that students were both matched and mismatched on learning style preference. Equal numbers of high-, medium- and low-ability students were assigned to each group.

The two treatments were delineated in the following manner. For both groups the teacher spent part of each class period reviewing previous material and introducing new material to the group as a whole. The content taught was a nine-day geometry unit from Developing Mathematical Processes. This was followed by approximately one-half of each 40-minute period with the students doing workbook assignments. Those in the large-group approach worked individually, asking questions of the teacher as necessary. Those who were assigned to the small-group approach worked in groups of four (one high ability, two middle ability, and one low ability). The students were to get assistance within the group and only as a last resort turn to the teacher.

Observations were conducted on each class to assess the implementation of the correct instructional process and to assess the group processes present.

At the close of the instruction, a geometry achievement test was administered along with a re-administration of the attitude toward teaching approach scales and the attitude toward mathematics scale. Two weeks later a retention test (geometry content) was administered. Generalized regression analyses were then performed on the achievement, attitude, and retention scores. Aptitudes, teacher, treatments, and aptitude-treatment interaction terms were entered into the equations.

4. Findings

For both the achievement data and the retention data, the only significant effects were ability and the curvilinear aptitude-treatment interaction for ability. This interaction showed that both high-ability and low-ability children learned best in a small-group approach while the teaching approach had no effect on the achievement of middle-ability children.

In the regression analysis of the attitude toward mathematics post-test data, the primary predictor was the attitude toward mathematics pre-test/locus of control factor. Ability was also a significant predictor, with low-ability students having more positive attitudes toward mathematics than high-ability students.

The attitude toward teaching approach data analysis yielded significant predictors of treatment, ability by treatment, and attitude/locus of control by treatment.

An analysis of the data on observations of group processes showed a significant relationship between the number of explanations given and achievement. This was not significantly related to attitude. Receiving explanations was not significantly related to achievement or attitude. High-ability students gave the most explanations, low-ability next, and middle-ability the least.

5. Interpretations

The authors state that their findings support the existence of a curvilinear aptitude treatment interaction for ability in such large-group small-group studies. An underlying factor here is when students teach students. No confirmation could be given on whether or not one should match preferred learning style with teaching approach.

Abstractor's Comments

The findings of the curvilinear aptitude-treatment interaction raises several questions. Could the large middle group's achievement be affected by work to improve their self confidence so that they might offer more explanations? Where is the best place to "cut" high and low ability to find such an interaction? What would happen if the groups only contained high- and low-ability students?

The authors do not really attempt to relate their study to actual classroom use, but rather discuss the research methodology. I feel that the finding which teachers might most apply is that of the advantages of students teaching students. Many experienced teachers will comment that the first time that they really understood some item of content was when they had to teach it. They may have received many explanations but now they had to give an explanation. Encouraging this in the classroom seems fruitful.

There may be a confounding of the findings of this study with the content. At first glance this study almost seems independent of content. The geometry seems to be a vehicle. However, it would seem that the inquiry, discovery approach of DMP would not lend itself well to the purity of the large-group approach. I think most teachers would want to allow on-task student discussion here.

Since social relationships frequently affect group interactions, a third treatment in which the children pick their own group would be an interesting addition. Other factors would then need to be controlled statistically. An additional extension would be to attempt a replication at maybe ninth-grade level to see if low-ability students still offer more explanations than middle-ability students.

Schofield, Hilary L. TEACHER EFFECTS ON COGNITIVE AND AFFECTIVE PUPIL OUTCOMES IN ELEMENTARY SCHOOL MATHEMATICS. Journal of Educational Psychology 73: 462-471; August 1981.

Abstract and comments prepared for I.M.E. by HAROLD L. SCHOEN, University of Iowa.

1. Purpose

To determine the relationships among teachers' mathematical achievement and attitudes and those of their students in grades 4 to 6.

2. Rationale

Recent teacher effectiveness literature suggests that the promotion of academic skills may not be compatible with the promotion of favorable student attitudes. Yet in the elementary mathematics teaching literature, it is generally held, in spite of little empirical support, that teachers who like the subject and are good at it will be likely to produce students with similar attributes. Furthermore, teachers lacking in these two traits are likely to foster students with similar deficiencies. The present investigation was designed to assess the validity of each of these apparently conflicting arguments.

3. Research Design and Procedures

Tests measuring mathematics achievement and attitudes toward mathematics and its teaching were administered to 251 prospective elementary school teachers (189 females and 62 males) from two Australian teachers' colleges toward the end of their final year of training. Of these teachers, the 56 who were assigned to classes in grades 4 to 6 were asked to administer tests of mathematics attitude and achievement to all their pupils toward the end of Term 1 (April) and again at the end of Term 3 (October). Based on the maximum number of pupils enrolled in the classes and on those teachers and pupils who participated in testing in both April and October, data came from 1,025 children (501 girls and 524 boys) in the classes of 50 teachers (30 female and 20 male).

The achievement test for teachers covered a broad range of mathematical concepts, and the attitude measure assessed the teachers' attitudes toward mathematics and its teaching. One student achievement test measured concept acquisition and the other measured computation skills.

One-way analysis of variance and covariance were used to investigate the relationship between teacher variables (at 3 levels--low, medium, and high) and pupil mathematics attitudes and achievement measures.

4. Findings

Compared with pupils of middle- and low-achieving teachers, the pupils of high-achieving teachers consistently exhibited the highest mathematics achievement on both the concepts test and the computation test in April and again in October. At the same time, these pupils exhibited significantly less favorable attitudes towards mathematics than the pupils of low- and middle-achieving teachers on all five attitude measures in April and all four that were significant in October. (The probability levels that are reported are $p < .01$ or less, with one .05 level difference noted.)

With respect to teacher attitudes toward mathematics, in comparison with middle- and low-attitude teachers, high-attitude teachers had consistently higher achieving pupils on both the concepts and computation tests at both testing times. At the same time, these pupils had significantly less favorable attitudes than the pupils of middle- and low-attitude teachers.

With respect to the teachers' attitudes toward "mathematics teaching," there was a consistent, positive relationship between the teachers' attitudes toward teaching mathematics and pupil mathematics achievement on both the mathematical concepts and computation tests at both testing times. The relationship between teacher attitude toward mathematics teaching and pupil attitude toward mathematics was slight.

Changes in teacher-pupil relationships between April and October were investigated using one-way analyses of covariance. In the main, the negative relationship between teacher attitude and pupil attitude strengthened with time. On the other hand, the relationship between teacher mathematics achievement and pupil mathematics achievement (computation), which was significant and positive in both April and October, reversed direction in this analysis; that is, the relationship became weaker over time. No other effects for teacher mathematics achievement or attitude on pupil mathematics attitudes and achievement evident in the October data were significant in the covariate analysis.

5. Interpretations

The present findings offer clear support for the commonly assumed positive association between teachers' attitudes and achievement in mathematics and pupil achievement in mathematics, but they do not support the contention that this association is achieved via pupil attitudes. Rather, there is support for the position that teacher behaviors that enhance the acquisition of mathematical skills may conflict with those that enhance the development of favorable attitudes toward the subject. Conversely, these results oppose both elements of the contention that the promotion in children of favorable attitudes toward mathematics is necessary for their adequate mastery of mathematical concepts and skills and that teachers who do not themselves possess the desired attitudes will be unable to transmit these attitudes to their pupils.

Abstractor's Comments

This study is a technically sound correlational study. However, it seems possible to view the results in ways quite different from the researcher's interpretation. For example, perhaps the prospective teachers with high scores on the achievement and attitude measures in this study attained more "desirable" teaching positions than did those who scored in the middle and low range. Since these measures, especially the achievement measure, very likely correlate positively with general intelligence and SES, this seems like a quite plausible expectation. More "desirable" teaching positions often mean brighter, more critical pupils. The relationships discovered in this study could then be the result of these initial student differences and would, of course, have nothing to do with the effect that teachers with pre-existing attitudes and achievement levels may have on their pupils.

Is this scenario more or less plausible than the researcher's interpretation? How seriously should we take the researcher's conclusions? Each interested reader must decide the answers to these questions for himself or herself. At least until further evidence is in, this mathematics teacher educator is not ready to try to develop teachers with negative attitudes toward mathematics in order to assure positive attitudes for their pupils.

Smead, Valerie S. and Chase, Clinton I. STUDENT EXPECTATIONS AS THEY RELATE TO ACHIEVEMENT IN EIGHTH GRADE MATHEMATICS. Journal of Educational Research 75: 115-120; November/December 1981.

Abstract and comments prepared for I.M.E. by BARBARA J. PENCE, San Jose State University.

1. Purpose

The purpose was to document and investigate the relationship between student expectations for achievement in mathematics and subsequent achievement in mathematics. Student expectations were in no way varied or changed experimentally. Student expectations included the perception of the individual's achievement and also sex-related achievement expectations.

2. Rationale

Study of self-fulfilling prophecies and the pygmalion effect has produced a significant body of experimental literature. Results generally support the relation between expectations and human performance.

Following their review of this literature, the authors noted that the classroom teacher was typically the focus for the independent variable. In studies where student expectations were explored, the expectations were artifically varied or experimentally induced. Consequently, the authors felt that further study related to the pygmalion effect required exploration of the relationship between actual student expectations and achievement. They hypothesized that, with general ability controlled, achievement would be directly related to the individual's expectations for achievement and also to the students' expectations for their sex-group's achievement in mathematics.

3. Research Design and Procedures

The sample consisted of 698 eighth-grade students from three schools in a southern Indiana city. The design related responses on a questionnaire with scores on two mathematics tests.

The questionnaire, developed by the experimenters and administered in October, elicited student expectations through their responses to seven items. Items one and two asked how well the students expected to do and what grades they expected in the mathematics class. Three questions explored parent and peer expectations relative to the students' mathematics achievement. The remaining two items asked for their sex and who usually did better in mathematics--boys, girls, or neither.

Student achievement expectations were classified into two levels, high or low. Students who expected a grade of A or B were assigned to the high-level group while students with expectations for a C, D, or F were assigned to the low-expectation group. Sex-role stereotypes were assigned three values paralleling the three responses favoring same sex, different sex, or expecting neither sex to achieve better.

Performance tests included two achievement tests and one test of mental ability. The first achievement measure was the total mathematics grade equivalent obtained on the combined Mathematics Skills and Concept and Problems subscales of the Iowa Test of Basic Skills (ITBS). The second criterion measure was the total score on a 30-item experimenter-compiled mathematics test (ECMT). Stanine scores on the cognitive Abilities Test (CAT) for the verbal, quantitative and nonverbal scales provided measures of mental ability. The ITBS and CAT were administered in December while the ECMT was administered in late April.

4. Findings

Data description included means and standard deviations for the ITBS, ECMT, and CAT divided according to the sex of the student, achievement expectation level (high, low), and perceived sex advantage (same sex, opposite sex, neither).

The basic analysis was a 2x3x2 analysis of covariance, student achievement expectation level by perceived sex advantage by sex. Each of the three subscales of the CAT served as covariates. Cell frequencies were equalized through random deletions. The ANCOVA was replicated for both the ITBS and ECMT scores. The ITBS analysis resulted in a significant relationship between expected achievement and achievement ($p < .01$). Data from the ECMT produced the same result at the same level of significance. No significance was found on either run between sex and achievement or between perceived sex advantages and achievement.

Additional tables described the achievement expectations by sex;

related the achievement expectations to the perceived expectations of others; and provided a profile of sex vs. sex role expectation relationships. A survey of this descriptive data yielded interesting results, such as: 86% of the high-expectancy group, but only 56% of the low-expectancy group, reported similar peer and parent expectations; those students with high expectations accounted for 69% of the sample; and significantly more girls stated high expectations for their sex ($p < .005$).

5. Interpretations

Student expectations significantly relate to subsequent achievement. Practically, it appears that a profitable classroom strategy would be to build confidence. The low-expectation students should be helped to believe that they will succeed.

The lack of significant relationships between sex perceptions and achievement was explained by the idea that the individual's achievement expectation out-weighed one's expectations for their sex group.

The descriptive data raised issues for further study. High achievement expectations were expressed by a large percentage of the students. Why does this overestimation continue through the elementary grades? Also, why do girls hold higher expectations for themselves when boys are believed to excel in eighth-grade mathematics?

Abstractor's Comments

Findings of this study were predictable. Indeed, the significant relationship between self-concept towards achievement in mathematics and subsequent actual mathematics performance has been documented. One major study which contributed to this area of knowledge was the National Longitudinal Study of Mathematical Ability (NLSMA) -- a large (over 112,000 students from 1,500 schools in 40 states), long-term (following specific populations of students up to five years) study which began testing in 1962. Results from the NLSMA study were published in 32 volumes and are available from ERIC. In NLSMA Report No. 20, Crosswhite described the attitude results and in NLSMA Report No. 27, Begle examined variables measured at one time which predicted mathematics achievement at another time.

Correlational results from NLSMA Report No. 20 support the major finding of this study. In grade 8, actual self-concept correlated with Spring measures of computation and structure at levels of .34 and .35, respectively. Crosswhite (p. 17) summarizes the data for the total study by stating, "in no case is the correlation large but the consistency strongly suggests a stable, positive relationship between attitude and achievement." As a result, the eighth-grade data can be viewed as representative of relationships existing across grades. The consistency of the relationship is stated to be not only stable across grades but also across scales of achievement. Thus, although it is usually risky to collapse performance on computation and application of concepts, in this case the analysis using the single combined score probably gave as much information as separate analyses.

Begle in Critical Variables in Mathematics Education also supports the major finding of this study in the statement that "there is a significant positive correlation between attitudes and achievement" (p. 87). He continues by stating that "the relationship, however, is not as strong as many seem to believe" (p. 87). It is interesting to note that in Begle's review of predictors of eighth-grade mathematics achievement, attitude scales never appeared as significant predictors either in the case when only psychological predictors were used or when both mathematics predictors and psychological predictors were merged. In his summary, Begle states that "the best predictors of mathematics achievement are usually previous mathematics achievement" (p. V).

Consequently, although the results are predictable, the implications cannot be justified on the basis of the findings. Significant positive relations between student achievement expectations and subsequent achievement fail to establish causality and certainly in no way define a direction for the relationship. To illustrate, Crosswhite (p. 17) points out that correlations with attitude scales are essentially the same whether the achievement is obtained the Spring before or the Spring after administration of the attitude measure. This lack of directional causality, however, does not and cannot deny the basic value system support for helping a student believe he or she can succeed.

Work for related studies not only helps with prediction of and interpretation of results; it also yields ideas concerning the instrumentation. It is indeed sad that the substantial literature in Mathematics Education on the relationship between achievement and expectations as well as sex-related expectations was not referenced. Certainly a more detailed understanding of mathematics expectations or self-concepts and their relationship to achievement would be helpful. One contribution in this area would be a standardization of measures or even a collection of measures. It is sad when experimental data are collected and then not analyzed because of a concern over reliability. It is also sad when experimenter-constructed measures are used, and there are no reliability statistics established on the instrument. This does not contribute to future work.

Many questions evolved as I worked through this study. Since my major concern was the lack of reference to related literature, I will explore only two of my questions.

First, although the combination of achievement scales for the investigation of expectation and achievement relationships made sense and could be defended, separation of the data into two scales of "computation" and "understanding" could have thrown some light on the sex-related correlations. Generally, by eighth grade, girls excel in computations while boys excel in the conceptual and problem-solving areas of mathematics.

Second, those students who reported inconsistent achievement expectations raised several questions. That is, further examination of those students who expected to receive either an A or B grade and described their general achievement as okay or bad or the reverse mismatch would be interesting. What were they really saying? If serious, could they have been responding on two different levels, such as the grade prediction reflected the actual expectation and the evaluation of work reflected an ideal expectation? Were any follow-up conferences conducted? If the answers were serious and the students were using similar references for their answers, were there enough such cases to do additional analyses?

In summation, the experimental questions addresss an issue where more work is needed. Documentation of relationships exist but an understanding of these relationships and ways in which to change self-concepts or self-expectations would be a significant contribution both theoretically and practically.

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Squire, Barry F.; Cathcart, W. George.; and Worth, Joan E. EFFECT OF MODE OF INSTRUCTION ON PROSPECTIVE TEACHERS' ATTITUDES TOWARD MATHEMATICS. Alberta Journal of Educational Research 27: 35-45; March 1981.

Abstract and comments prepared for I.M.E. by LARRY K. SOWDER, Northern Illinois University.

1. Purpose

The primary focus of this study was an examination of the relative influence on preservice elementary teachers' attitudes of two class formats, the "conventional" lecture setting and a "seminar-workshop approach."

2. Rationale

Teacher attitude is usually regarded as influential in shaping student attitude. There is some evidence that using a laboratory approach has a positive effect on preservice teacher attitude.

3. Research Design and Procedures

Five sections of a semester course in elementary mathematics curriculum and instruction provided the 269 subjects, with 174 in the "conventional" lecture treatment (three lectures a week). The other 95 students formed the experimental groups, for which the instruction consisted of one lecture, one laboratory session, and one seminar a week. The five sections were taught by five instructors.

Three measures were given in a pre-post design: (a) ratings on four items relating enjoyment, worthwhileness, and competence to teaching four major high school subject areas; (b) an item from the Dutton scale to measure the students' self-appraisals of their general feelings toward mathematics; and (c) Aiken's 1972 attitude scale (Likert, 20 items).

4. Findings

Students with higher attitude scores tended to rate mathematics more favorably on the rating items--(a) above.

With $\alpha = 0.05$, the posttest score on the Aiken scale was statistically superior to the pretest score only for the (grouped) experimental sections (55.9 to 64.8). Blocking on pretest Dutton scores into three levels revealed an interaction between treatment and pretest level: at the lowest pretest level, the lecture method was superior to the seminar approach, with the reverse holding for the middle level and with virtually no difference at the upper level.

5. Interpretations

The seminar-workshop approach seemed to enhance attitudes more than the lecture method; this finding supports the bias of many mathematics educators. The interaction noted above, however, suggests that students with the lowest attitude scores initially profit more from the lecture method.

Abstractor's Comments

Like the authors, I believe that improving preservice teachers' attitudes toward mathematics is important. It is only marginally clear, however, whey they thought the difference in class format would make a difference. Was it that the seminar-workshop format allowed for laboratory approaches? If so, two of the four small sections could have been given laboratory experiences and the other two none. As the study was carried out, class size and class format are confounded. One might conclude just as well that smaller classes, not the format, improve attitudes more.

The use of multiple measures of attitudes is an excellent feature of the study, particularly in an area of measurement as "soft" as attitude measurement. It is not clear why only one item was used from the Dutton scale, nor why no correlation with the Aiken scale is reported.

Assignment of student to treatment was not under the experimenter's control, and they note the proper caveats. It was surprising that they did not comment on the use of five instructors. Perhaps some degree of uniformity was built into the study; otherwise any of the results could be attributed to instructor effects.

The study does have flaws in rationale and design. Perhaps one of its better uses would be as a subject for discussion in a research-critique course.

Vest, Floyd. COLLEGE STUDENTS' COMPREHENSION OF CONJUNCTION AND DISJUNCTION. Journal for Research in Mathematics Education 12: 212-219; May 1981.

Abstract and comments prepared for I.M.E. by LARS C. JANSSON, University of Manitoba.

1. Purpose

The purpose of the study was to determine the level of college students' comprehension of conjunction and disjunction and to determine if invalid inference patterns are followed by nonnegligible proportions of students.

2. Rationale

It has been hypothesized that some people incorrectly use the common logical connectives because they consistently apply incorrect logical rules and thus draw incorrect conclusions. "In a recent study of college students' comprehension of implication, O'Brien (1973) observed that students performed below the chance levels on certain subtests, and he located and described incorrect inference patterns that were consistently followed by 'nonnegligible' proportions of the students." The present study extended this research into the area of conjunction and disjunction using methods similar to those of O'Brien.

3. Research Design and Procedures

An untimed test of 32 multiple-choice items was administered to a sample of 115 first-year college students who were nonscience majors. A sample item is given below:

Given: (Mary is in Room 5 or she is in the first grade) is False.

Given: (Mary is in Room 5) is False.

(1) (Mary is in the first grade) is True.

(2) (Mary is in the first grade) is False.

(3) The given conditions are inconsistent.

(4) No one of the above alternatives is valid.

Thus, in the set of items the truth of the first two premises was varied to give the four combinations TT, TF, FT, FF. The example above was coded FF. Each of these pairs was labeled a "component" and each component contained four items. The same format was used for conjunction and disjunction, resulting in the 32-item instrument. In each of these two subtests, only two basic simple sentences were used. Each alternative 1, 2, 3, 4 was a correct response for four test items in each subtest. "For the test construction, the order of individual items was randomized subject to the constraint that no single correct alternative response occurred more than twice in sequence. The same sequence of test items was presented to all students."

4. Findings

For the conjunction subtest, 86.7% of the responses to the TT component were correct, but the level of success was substantially lower for the other three components, and "several incorrect responses were chosen more often than expected by chance." It was hypothesized that "certain inference patterns may be followed consistently by significant proportions of the students...(and it was specified) that an inference pattern is followed consistently...in a four-item component when the same alternative response is given three or more times." For components TF, FT, and FF, substantial proportions of the subjects responded according to invalid conjunction inference patterns. For example, for the FF component, "66.1% of the students consistently responded according to the invalid rule: If in the given condition, (P and Q) is false and one of the simple sentences is false, then the other simple sentence is false."

For the subtest on disjunction, 70.9% of the responses for the TF component were correct, and for the FF component 57.8% were correct. "For the TT and FT components, incorrect responses occurred more often than expected by chance. The actual frequencies were compared to the expected using the normal approximation to the binomial distribution. "The majority of the responses to the TT component followed the invalid inference pattern: If (P or Q) is true and one of the simple statements is true, then the other simple statement is true. The majority of responses to the FT component followed the invalid pattern: If

(P or Q) is false and one of the simple statements is true, then the other simple statement is false." Large proportions of students consistently followed invalid patterns within these two components.

5. Interpretations

The majority of the students did not comprehend conjunction and disjunction well enough to respond correctly to most of the components of the subtest. Several error patterns were identified that were frequently and consistently followed by large proportions of students. "Two frequently followed error patterns for conjunction suggest that students consistently interpret (P and Q) is false as (P is false and Q is false). ...From these results it is concluded that certain college students consistently use interpretations of conjunction and disjunction that are different from the definitions given in logic."

Abstractor's Comments

The study is an extension of O'Brien's (1973) work and thus employs the same type of items. Both researchers are to be commended for this item type and its use of a "given conditions are inconsistent" alternative.

The search for patterns of reasoning is a laudable goal which has potential payoffs for teachers, although the number of such patterns may turn out to be more than just a few, and with small numbers of students following each. This should be the subject of continuing investigation.

All items in each subject employed the same first two simple sentences as premises, with the truth values varied to construct the four components. In this format can students actually treat each item independently of the preceeding items? There is no evidence that students were informed of this independence, despite otherwise good explanatory instructions. However, even if they were told it, is it possible to actually carry out the reasoning on each item independently, or does the cognitive overload become too great?

Despite the very positive contribution of this study, it is unfortunate that the investigator has not provided more reference to previous studies. Much more has been done in this area than he suggests and the articles are far too numerous to mention here.

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